Clinical Anatomy 18:126–130 (2005)

ORIGINAL COMMUNICATION

Simple Formula for the Surface Area of the Body and a Simple Model for Anthropometry

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The body surface area (BSA) of any adult, when derived from the arithmetic mean of the different values calculated from four independent accepted formulae, can be expressed accurately in Système International d'Unités (SI) units by the simple equation $BSA = 1/6(WH)^{0.5}$, where W is body weight in kg, H is body height in m, and BSA is in m^2. This formula, which is derived in part by modeling the body as a simple solid of revolution or a prolate spheroid (i.e., a stretched ellipsoid of revolution) gives students, teachers, and clinicians a simple rule for the rapid estimation of surface area using rational units. The formula was tested independently for human subjects by using it to predict body volume and then comparing this prediction against the actual volume measured by Archimedes' principle. Clin. Anat. 18:126–130, 2005.

Key words: human; anthropometric; weight; height; spheroid; ellipsoid

INTRODUCTION

The surface area of the human body is an important measure of body size in physiology and clinical medicine. In particular, body surface area (BSA) has been employed in scientific studies and clinical practice to standardize various measurements concerning body heat transfer, body aerodynamics and hydrodynamics in sport, renal function, body metabolism, chemotherapy, and cardiac function (Norton and Olds, 1996; Yu et al., 2003).

Several attempts have been made to compute the body surface area from simple measurements made on the living body (anthropometric data). Since Du Bois and Du Bois (1916) first presented their formula to determine BSA, there have been three other major, independent investigations, each of which has resulted in a different formula of similar mathematical construction using the same variables of body weight (W) and body height (H). These formulae are:

i. $BSA = 71.84W^{0.425}H^{0.725}$ (Du Bois and Du Bois, 1916)
ii. $BSA = 178.7W^{0.4838}H^{0.500}$ (Boyd, 1935)
iii. $BSA = 235W^{0.51456}H^{0.42246}$ (Gehan and George, 1970)
iv. $BSA = 242.65W^{0.5378}H^{0.3964}$ (Haycock et al., 1978)

where W is the body weight in kg, H is the height in cm, and BSA is the body surface area in cm^2.

In all cases, the same formula applies for either gender. Applied to a single individual, say a person of 70 kg whose height is 175 cm, the formulae yield the following surface areas: 18,481 cm^2 (i), 18,463 cm^2 (ii), 18,539 cm^2 (iii), and 18,468 cm^2 (iv). It can be seen that these values vary from one another by no more than 0.4%. It could be argued that, in the face of such excellent agreement in values, there is little point in developing a fifth formula of BSA. For the anatomist, physiologist, or clinician who requires a rapid estimate of surface area, however, none of the above formulae are intuitive and each is difficult to memorize and time-consuming to evaluate.

Our aim, therefore, was to attempt a derivation of the surface area of the human body that would lead to a rapid and reasonable 'rule-of-thumb' for body surface area in rational units of the Système International d'Unités (SI).

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Received 29 January 2004; Accepted 14 April 2004

Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/ca.20047

METHODS

We first calculate the arithmetic mean of BSA from the four separate formulae above and designate this as the mean body surface area (MBSA). This mean is a more accurate physiological measure for clinical purposes because (i) the four formulae have all been derived by independent studies and the mean of any available independent estimates is the best measure (Bland and Altman, 1986) and because (ii) there is no yardstick or 'gold standard' against which one can determine the accuracy of any one formula. For the example above (weight = 70 kg, height = 175 cm), the mean body surface area (MBSA) based on the four formulae is 18,488 cm^2.

Anthropometric values were obtained from the senior author's databank based on 257 male and 247 female adults (Reading, 1996). The measurements were compiled from a sample of patients attending a suburban Sydney clinic for routine medical reasons over several years; all subjects were normal Australian adults. The age range of female subjects was 25.00–83.06 years (mean = 43.36 years; SD = 13.10 years) and the age range of the male subjects was 25.05–77.52 years (mean = 45.14 years; SD = 13.82 years). Each subject was weighed wearing only underwear and no allowance was made for state of fullness of bowel or bladder. Height was measured without shoes. Using the four standard formulae, the four BSA values were calculated for each person and then the mean (MBSA) was determined. Next, using the same height and weight data, the surface area of a body-equivalent ellipsoid was also calculated for each individual. The rationale for modeling the body as an ellipsoid is described below.

An important aspect of this study was to investigate whether the human body could be modeled by a simple geometric figure, in the hope that this would then yield a less cumbersome formula for body surface area. Boyd (1935) has summarized previous attempts to represent the human body geometrically as a rectangular prism, or as a right cylinder, or even as a series of truncated cones. Because the surface of the body is nowhere planar, but for the most part represents a series of surfaces of gradually changing curvature, it was decided to model the body as an elongated (stretched) ellipsoid. An ellipsoid has three principal semi-axes of length a, b, and c in the x, y, and z axes, respectively; if a = b = c, the ellipsoid is a sphere; if two semi-axes are of equal length, then it is an ellipsoid of revolution or spheroid. We consider the specific case where the third semi-axis is longer (major semi-axis) than the two equal semi-axes (minor semi-axes). This is a symmetrical solid, derived by rotation of an ellipse around the major axis; its shape is similar to that of a rugby football (Fig. 1) and it is designated a prolate spheroid or stretched ellipsoid of revolution. The volume (V) of the prolate spheroid is $(4\pi a^2 c)/3$ and its surface area (SA) is $4\pi ac$, where a = b is the minor semi-axis and c is the major semi-axis. This solid figure will be designated the body-equivalent ellipsoid (BE).

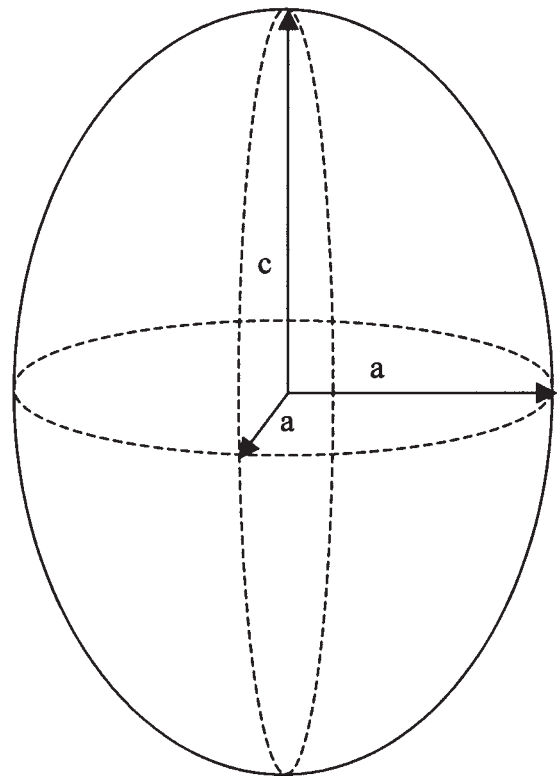

Fig. 1. The prolate spheroid (stretched ellipsoid of revolution) with equal minor semi-axes of length a and major semi-axis of length c.

Our hypothesis is that the surface area of the BE is proportional to the actual body surface area. If the BE is equivalent to a vertical human body and if the length of the major semi-axis (c) is represented by half the body height (H/2), then the two equal minor semi-axes (of unknown value) will lie in the mid-horizontal plane. All horizontal transverse sections will be circular in shape and all vertical sections will be elliptical. As all four computed formulae for BSA are claimed to apply to either gender, it is apparent that variations in body density between males and females can be ignored. Body density will be assumed initially to be 1 g/cm^3, and thus the magnitude of body weight will equal body volume. Typical values of actual body density are 1.065–1.075 g/cm^3 (Vickery et al., 1988). Because body mass is the product of density and volume, if the density is underestimated at 1 g/cm^3, then the body volume will be overestimated by only

TABLE 1. Statistical Data for Proportionality Factor *k* where *k* = MBSA/BESA

	n	Maximum *k*	Minimum *k*	Mean *k*	SD
Males	257	1.2203	1.2099	1.2153	0.0016
Females	247	1.2213	1.2086	1.2165	0.0023
Combined	504	1.2213	1.2086	1.2158	0.0020

6.5–7.5%. For a body-equivalent ellipsoid of height H and minor semi-axis a:

$$\text{Body weight} \cong \text{BE volume (BEV)} = (4\pi a^2 H)/6 \tag{1}$$

$$\text{BE surface area (BESA)} = 2\pi a H \tag{2}$$

For H in cm and W in kg (strictly kg. wt.), equation (1) simplifies to weight (gm) = 1000 W (kg) = $2.0944a^2H$, which then yields an expression for the unknown minor semi-axis $a = 21.851(W/H)^{0.5}$. On substituting this value for a in equation (2), BESA = $137.3(W \times H)^{0.5}$, where BESA is in cm^2, H is in cm, and W is in kg. For the case of a 70 kg, 175 cm high person, the surface area of the body-equivalent ellipsoid (BESA) is 15,196 cm^2. Under our working hypothesis, the surface area of this simple solid is proportional to the actual body surface area, i.e., MBSA = kBESA where k is a proportionality factor. In the above example, $k = 1.2166$.

RESULTS

Utilizing the databank of adult body measurements (Reading, 1996), both MBSA and BESA were calculated for each individual. The MBSA for each person was then divided by the calculated BESA, to yield the individual value for the proportionality factor k_i. The ranges and mean values for k_i obtained from the sample population of 257 males and 247 females are summarized in Table 1.

It can be seen from Table 1 that the individual values for k_i exhibit very little scatter, irrespective of gender. The mean value (k_m) for the proportionality factor for all 504 subjects is 1.2158. The combined standard error (SE) of this mean is 0.000089, which is negligible compared to k_m, indicating that $k = 1.2158$ can be taken as a constant. Therefore, for either gender, MBSA may be obtained by multiplying the ellipsoid surface area by k, i.e., MBSA = kBESA = $1.2158 \times 137.3(WH)^{0.5} = 166.9(WH)^{0.5}$, where MBSA is in cm^2, W is in kg, and H is in cm. By converting to coherent SI units (H in meters, W in kg, BSA in m^2) and choosing a rational fraction closest in value to the multiplier, this equation for body surface area may be simplified to:

$$\text{BSA} = 1/6(\text{WH})^{0.5}. \tag{3}$$

In this equation, the percentage difference in value between the rational fraction of 1/6 and the calculated number (0.1669…) is only about 0.15%. This is almost the same formula as an empirical square root formula considered previously by both Du Bois and Du Bois (1916) and Gehan and George (1970), which was utilized subsequently by Mosteller (1987) and validated for the surface area of the child's body by Lam and Leung (1988).

The formula BSA = $1/6(WH)^{0.5}$ was tested independently as follows. If the prolate ellipsoid of revolution is a valid representation for a human body, then it should predict body volume. Conversely, body volume should predict the dimensions of the BE. By constructing a special total-body immersion bath joined to a volumetric measuring chamber by a wide-bore tube (Reading, 1996), it was possible to use Archimedes' principle to determine the body volume for five subjects. Based on the individual values for body volume (in m^3) and the related anthropometric measurements, the value of the equivalent horizontal minor semi-axis a (m) was calculated for each BE, using the equation $a = [6V/(4\pi H)]^{0.5}$, where V is body volume in m^3 and H is body height in m. When this value of a (derived from the independent volume data) was used to determine the surface area of the body-equivalent ellipsoid and this value was increased by the proportionality factor k (1.2158), the values were found to agree closely with the values of BSA derived from the simple formula above. The results are summarized in Table 2. Despite the small sample size, the low percentage difference (Δ%) between the two SA values provides an independent test for the body-equivalent ellipsoid hypothesis.

DISCUSSION

Our treatment of body surface area has followed two approaches. The first approach was to determine the mean of the values from four extant formulae for body surface area. In the absence of a 'gold standard' for surface area, this value, designated MBSA, is a more accurate estimate of the actual surface area. Our second approach was to hypothesize that the human body, in all its surface complexity, could be represented by a body-equivalent ellipsoid of revolution, and that the surface area of the model (a prolate spheroid) should then be proportional to MBSA.

TABLE 2. Body Surface Area Determined by Volumetric Measurement vs. Formula[a]

S	Gender	W (kg)	H (m)	V (m^3)	$a = [6V/(4\pi H)]^{0.5}$ (m)	BESA = $2\pi aH$ (m^2)	*k*BESA (*k* = 1.2158) (m^2)	BSA = $1/6(WH)^{0.5}$ (m^2)	Δ (%)
1	F	59.55	1.765	0.05945	0.1268	1.4062	1.7097	1.7087	−0.059
2	M	66.36	1.702	0.06475	0.1348	1.4415	1.7526	1.7713	1.056
3	F	73.18	1.746	0.07335	0.1416	1.5534	1.8886	1.8839	−0.250
4	F	59.00	1.651	0.05846	0.1300	1.3486	1.6396	1.6449	0.322
5	F	56.60	1.696	0.05590	0.1254	1.3363	1.6247	1.6329	0.502

[a]S, subject number; M, male; F, female; W, weight (kg strictly kg wt.); H, height (m); V, volume of body (m^3) determined by the method of Archimedes' principle; a, equivalent horizontal minor semi-axis (m); BESA, body-equivalent ellipsoid surface area, BSA, body surface area (m^2) obtained from simple formula BSA = $1/6(WH)^{0.5}$, Δ (%), percentage difference in SA values, i.e., (BSA - *k*BESA)/BSA × 100.

We showed that the individual values of the proportionality factor (k_i) have very little scatter. Furthermore, we showed that independent measurements of body volume could be used to predict a similar surface area, thereby substantiating the use of the ellipsoidal model. Therefore, it is apparent that the mean body surface area may be readily estimated in rational SI units from the relation BSA = $1/6(WH)^{0.5}$, which is a formula that is easy to memorize and evaluate. This formula is of unusual accuracy for a physiological equation.

The fact that our formula, based on the MBSA–BESA approach, is essentially identical to the formula utilized empirically by Mosteller (1987) is a further validation of the ellipsoid model for the human body. Perhaps even more remarkable is the similarity of our simple formula with the most recent formula for body surface area derived by the completely new and independent method of 3D one-pass whole-body scanning (Yu et al., 2003). The formula derived by Yu et al. (2003) can be expressed as BSA = $1/6.28(WH)^{0.5}$ using SI units. The difference between our coefficient of 1/6 and their coefficient of 1/6.28 is about 4%. This difference is well within the total percentage error computed by Yu et al. (2003) when they compared their results with the results of calculating body surface area using the older formulae described in our Introduction. It therefore seems possible to dispense with all previous cumbersome formulae for the estimation of body surface area. By making appropriate approximations to W and H, the calculation may be done rapidly without a calculator, e.g., for a person of weight 60 kg and height 1.7 m, the formula yields $1/6(102)^{0.5}$; by approximating with the square root of 100, the BSA is about 1.67 m^2, which is <1% from the calculated value.

We noted above the consequences of simplifying the calculations by underestimating body density at 1 g/cm^3. A more accurate estimate of BSA would take actual body density into account, as follows. Body-ellipsoid volume (BEV) = $(4\pi a^2 H)/6$ = body weight/body weight density, whence for weight W and weight density γ, $a = [6W/(4\gamma\pi H)]^{0.5}$. Substituting this value for a in the formulae for surface area yields BESA = $[(6\pi WH)/\gamma]^{0.5}$ and MBSA = $k[(6\pi WH)/\gamma]^{0.5}$. Dimensional analysis of this expression in terms of mass (M), length (L), and time (T) shows that *k* is dimensionless, because $[MLT^{-2}]$ represents the dimensions of body weight and $[ML^{-2}T^{-2}]$ the dimensions of weight density. On the other hand, by ignoring body density, our simple formula for surface area of the body can only be dimensionally exact if the factor 1/6 in the equation has the dimensions $[M^{-1/2}LT]$. In general, any formula of the general form SA = $\kappa W^x \times H^y$, where SA is represented dimensionally by $[L^2]$, κ by unknown dimensions of $[M^\alpha L^\beta T^\gamma]$, W by $[MLT^{-2}]$, and H by [L], can only be dimensionally exact when $\alpha = -x$, $\beta = 2 - (x + y)$, and $\gamma = 2\times$. Even if *k* is taken to be dimensionless and W is assumed, as by Gehan and George (1970), to have dimensions $[L^3]$, then the general equation SA = $\kappa W^x \times H^y$, which can then be expressed dimensionally as $[L^3]^x[L^1]^y = [L^2]$, can only be dimensionally exact if $3x + y = 2$. It should be noted that the formula for BSA derived by Du Bois and Du Bois (1916) and the simple formula above conform to this condition and are thus mathematically valid, whereas the other formulae vary slightly from a dimensionally exact expression.

ACKNOWLEDGMENT

This article is based on the MMed. dissertation prepared while B.D.R. was a postgraduate student in the Department of Anatomy, University of Sydney.

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